Business Analytics

Prof. Phil Jones

Fall 2016

**Exam #1**

**Instructions:**

1. Please write the name of everyone in your group

at the top of each page.

* The exam is open book, open note, and open computer.
* You are expected to do your work within your group alone.

Note: Problems are worth points as indicated.

Providing short explanations may allow awarding partial credit in some cases.

The exam is due **September 21**. Please return your exam (hardcopy) at the beginning of class. If you are unable to return your exam in class, you can submit via email, but please do so only if absolutely necessary.

*Good Luck!*

Names: **Jerry Jacob**

**Danen Sorenson**

**Purna Chandra Kuntla**

**Andrew Peterson**

**Patty Johnston (Met only in one class)**

1. (10 points) **A height problem:** Suppose that the height of women in the United States is normally distributed with a mean of 65 inches and std. deviation of 2.5 inches. Suppose you select a random sample of 200 women within the United States. Let N = the number of those women whose height is less than or equal to 62 inches.

* 1. (5 points) What kind of probability distribution does N follow? Specify the name and parameters of the distribution.

**Normal Distribution**

**Mean=65**

**Standard Deviation=2.5**

**P(N) = P(X<=62)**

**= NORMDIST(62, 65, 2.5, 1)**

**= 0.11507**

* 1. (5 points) What is the probability that N is less than or equal to 25?

**P(N<=25) = BINOMDIST(25, 200, 0.11507, 1)**

**= 0.715848965**

1. (20 points) **An urn problem:**  An opaque urn contains 30 red balls, 40 blue balls, and 30 green balls. Suppose you draw exactly twice from the urn (with replacement) so that the odds of a given color are the same on the second draw as they were on the first draw.

**Ball drawn with replacement: Independent Events**

**P(R) = 3/10 P(G) = 3/10 P(B) = 4/10**

**Outcomes with 2 draws = 9 outcomes**

**= RR, RG, RB, GR, GG, GB, BR, BG, BB**

1. (5 points) What is the probability that at least one ball is blue given that at least one ball is either green or red?

**A = At least one ball is blue.**

**B = One ball is either green or red.**

**P(AB) = Outcomes with one ball blue and the other ball either green or red**

**= P(RB) + P(GB) + P(BR) + P(BG)**

**= P(R)\*P(B) + P(G)\*P(B) + P(B)\*P(R) + P(B)\*P(G)**

**= 3/10\*4/10 + 3/10\*4/10 + 4/10\*3/10 + 4/10\*3/10**

**= (12 + 12 + 12 + 12)/100**

**= 12/25**

**P(B) = P(RR) + P(RG) + P(RB) + P(GR) + P(GG) + P(GB) + P(BR) +P(BG)**

**= (9+9+12+9+9+12+12+12)/100**

**= 21/25**

**P(A|B) = P(AB) / P(B) = 12/21**

**= 4/7**

1. (5 points) What is the probability that at least one ball is blue given that the second ball drawn is either green or red?

**A = At least one ball is blue.**

**B = Second ball is either green or red.**

**P(AB) = Outcomes with first ball blue and second ball either green or red.**

**= P(BR) + P(BG)**

**= P(B)\*P(R) + P(B)\*P(G)**

**= 4/10\*3/10 + 4/10\*3/10**

**= (12 + 12)/100**

**= 6/25**

**P(B) = P(RR) + P(RG) + P(BG) + P(BR) + P(GR) + P(GG)**

**= (9+9+12+12+9+9)/100**

**= 15/25**

**P(A|B) = P(AB) / P(B) = 6/15**

**= 2/5**

c. (5 points) What is the probability that at least one ball is blue given that at least one ball is either green or red? **Same as Question a = 4/7**

1. (5 points) What is the probability that at least one ball is blue given that the second ball drawn is either green or red? **Same as Question b = 2/5**

3. (10 points) **A Card Problem**: Suppose you draw from a randomly shuffled deck of 52 cards (4 suits, each with 13 cards numbered 2 through Ace) 5 times **without** replacement. Without replacement means that after each draw, the drawn card is removed from the deck and the remaining cards in the deck are re-shuffled before drawing again. Hint: how is this similar to the “multiple birthdate” problem gone over in class?

* 1. (3 points) What is the probability that you get a royal flush in spades? A royal flush in spades means that after drawing the five cards, you have the ace, king, queen, jack, and 10 of spades.

**= 5/52 \* 4/51 \* 3/50 \* 2/49 \* 1/48**

**= 0.0000003848**

* 1. (2 points) What is the probability you get a royal flush (in any of the four suits ---clubs, diamonds, hearts, or spades) ?

**= Probability of outcomes in the first pick is (5\*4)/52**

**= 20/52 \* 4/51 \* 3/50 \* 2/49 \* 1/48**

**= 0.00000153908**

* 1. (3 points) If one million people repeat this experiment using different but otherwise identical decks of 52 cards, what is the expected number that get a royal flush?

**= 1,000,000 \* 0.00000153908**

**= 1.53908**

* 1. (2 points) If one million people repeat this experiment using different but otherwise identical decks of 52 cards, what is probability that at least one person gets a royal flush?

**P(X>=1) = 1 – P(X<=0)**

**= 1 - BINOMDIST(0,1000000, 0.00000153908,1)**

**= 0.785422**

1. (25 points) **A World Series Problem:** Recall the world Series Problem from problem set #3. In the World Series, there are two baseball teams, one from the National League and one from the American League. The series ends when the first (and winning) team wins 4 games. Thus, the series cannot end in less than four games. Since each game is played until there is a winner (no ties allowed), the series cannot extend past 7 games. Also, assume that each game is independent of any other game---that is, the probability of any team winning any game is 50%, independent of the outcome of previously played games.
2. (5 points) Assume that the teams are evenly matched---that is, each team has an equal 50-50 chance of winning each game. In that case, what is the overall probability that the American League team wins the series?

**With 50-50 chance of winning the game,**

**Probability of American League winning the series is 50%**

1. (15 points) Now assume that the odds are in favor of the American League team: for each individual game, the probability that the American League team will win is 60% (which means that the National League team has a 40% chance of winning each individual game). Now, what is the overall probability that the American League

**American League could win in 4, 5, 6 or 7 games**

**P(Winning in 4 games) = BINOMDIST(4,4,0.6,0) = 0.1296**

**P(Winning in 5 games) = BINOMDIST(3,4,0.6,0) \* 0.6 = 0.20736**

**P(Winning in 6 games) = BINOMDIST(3,5,0.6,0) \* 0.6 = 0.20736**

**P(Winning in 7 games) = BINOMDIST(3,6,0.6,0) \* 0.6 = 0.165888**

**P(American League winning series) = Add all the probabilities**

**= 0.71 = 71%**

1. (5 points) For the original problem from problem set 3 (equal probabilities of winning each game), let N = # games actually played in the series. What are the mean and standard deviation of N?

**Any of the team could win in 4, 5, 6 or 7 game**

**Mean = E(X) = n \* p**

**= 4 \* 0.125 + 5 \* 0.25 + 6 \* 0.3125 + 7 \* 0.3125**

**= 5.8125**

**Variance = n \* p \* (1-p)**

**= 4 \* 0.125 \* (1-0.125) + 5 \* 0.25 \* (1-0.25) + 6 \* 0.3125 \* (1-0.3125) +**

**= 4.167969**

**Std. Dev = SQRT(4.167969)**

**= 2.04156**

5. (10 points) **Rattlesnakes and Prairie Dogs**: A youth baseball league in Wyoming plans to institute a playoff series between the championship teams from each of their two divisions: the prairie dog division and the rattlesnake division. Unlike professional baseball, games in the youth league are allowed to end in ties. Thus, there are three possible outcomes for any game: prairie dog team wins & rattlesnake team loses, prairie dog team loses and rattlesnake team wins, they tie. The youth league plans to play the series until three games with the same outcome have occurred. If there are three wins by one of the divisions, that division’s team will win the championship, but if there are three ties, the championship will be declared a tie. Assume that the outcomes of successive games are independent events, and that the probability of either team winning a specific game is 45% (the same for both teams) while the probability of a tie is 10%.

1. (2 points) Let N denote the number of games the series lasts. What is the maximum possible value for N? The minimum?

**N max = 7 N min = 3**

1. (4 points) What is the probability that the series lasts exactly 3 games?

**= P(prairiedogs win) + P(rattlesnakes win) + P(tie)**

**= BINOMDIST(3,3,0.45,0) + BINOMDIST(3,3,0.45,0) + BINOMDIST(3,3,0.10,0)**

**= 0.091125 + 0.091125 + 0.001**

**= 0.18325**

**= 18.33%**

1. (4 points) What is the probability that the series lasts exactly 4 games?

**= BINOMDIST(2,3,0.45,0) \* 0.45 +**

**BINOMDIST2,3,0.45,0) \* 0.45 +**

**BINOMDIST(2,3,0.10,0) \* 0.10**

**= 0.303413**

**= 30.34%**

1. (15 points) **Girls and Boys**. Suppose the gender of a newborn child is equally likely to be male or female (a 50% chance either way). Suppose further, that you know a couple with exactly four children (no twins or triplets or quadruplets). You may assume that the gender of successive births are independent events.

**16 Outcomes = BBGG BBGB BBBB BBBG**

**GGGG GGGB GGBB GGBG**

**GBBB BGBB GBGG BGGG**

**GBBG GBGB BGBG BGGB**

1. (5 points) What is the probability that at least two of the children are girls given that at least one is a boy?

**A = At least 2 children are girls.**

**B = One child is a boy.**

**P(AB) = 10/16**

**P(B) = 15/16**

**P(A|B) = 2/3**

1. (5 points) What is the probability that at exactly two of the children are girls given that the youngest child is a girl?

**A = Exactly 2 children are girls.**

**B = Youngest child is a girl.**

**P(AB) = 3/16**

**P(B) = 8/16**

**P(A|B) = 3/8**

1. (5 points) What is the probability that at exactly two of the children are girls given that at least one of the children is a girl?

**A = Exactly 2 children are girls.**

**B = One child is a girl.**

**P(AB) = 6/16**

**P(B) = 15/16**

**P(A|B) = 2/5**

1. (10 points) **Sickle Cell Children.** Consider a population consisting of 1,000,000 adult females and 1,000,000 adult males. In this population, 10,000 females and 10,000 males code SS for the Sickle Cell gene, 100,000 of each gender code SA for the sickle cell gene, and the remainder code AA. Suppose that all of the people in this population eventually marry someone of the opposite gender and suppose that the pairing is done randomly (at least as far as the sickle cell gene goes). That is, let us consider a random male---since 1% of the females code SS, 10% code SA, and 89% code AA, then there is a 1% chance that this male will wind up marrying a female who codes SS, a 10% chance that the male will wind up marrying a female who codes SA, and an 89% chance that the male will wind up marrying a female who codes AA. Suppose that each of the 1,000,000 resulting couples has two children so that there will be 2,000,000 children born. In this new generation, what will be the expected numbers of offspring coding SA, AA, and SS? Would we expect the number fall into each category in the new generation to rise, lower, or stay the same?

**SS-SS: 0.0001 (Couple probability)**

**SS: 0.0001 = 200 (Offsprings)**

**SS-SA: 0.001**

**SS: 0.0005 = 1000**

**SA: 0.0005 = 1000**

**SS-AA: 0.0089**

**SA: 0.0089 = 17,800**

**SA-SS: 0.001**

**SS: 0.0005 = 1000**

**SA: 0.0005 = 1000**

**SA-SA: 0.01**

**SS: 0.00025 = 5,000**

**SA: 0.00025 = 5,000**

**AS: 0.00025 = 5,000**

**AA: 0.00025 = 5,000**

**SA-AA: 0.089**

**AA: 0.0445 = 89,000**

**SA: 0.0445 = 89,000**

**AA-SS: 0.0089**

**SA: 0.0089 = 17,800**

**AA-SA: 0.089**

**AA: 0.0445 = 89,000**

**SA: 0.0445 = 89,000**

**AA-AA: 0.7921**

**AA: 0.7921 = 1,584,200**

**Totals**

**SS: 200 + 1000 + 1000 + 5,000 = 7,200 (0.36%)**

**AA: 1,584,200 + 89,000 + 89,000 + 5,000 = 1,767,200 (88.36%)**

**SA: 1000 + 17,800 + 1000 + 10,000 + 89,000 + 8,900 + 89,000 = 225,600 (11.28%)**

**SS decreased by dilution from 1% to 0.36%**

**AA decreased by dilution from 89% to 88.36%**

**SA increased from 10% to 11.28%**